Paper Reference(s)

6677/01

Edexcel GCE

Mechanics M2

Advanced Level

Thursday 31 May 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40690A

1. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

A particle P moves in such a way that its velocity $\mathbf{v}$ m s^{-1} at time t seconds is given by

$$\mathbf{v} = (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}.$$

(*a*) Find the magnitude of the acceleration of P when $t = 1$.

(5)

Given that, when $t = 0$, the position vector of P is $\mathbf{i}$ metres,

(*b*) find the position vector of P when $t = 3$.

(5)

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2. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass $4m$ moving on the plane with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(*a*) Find the speed of Q immediately after the collision.

(6)

Given that the direction of motion of P is reversed by the collision,

(*b*) find the range of possible values of e.

(5)

3.

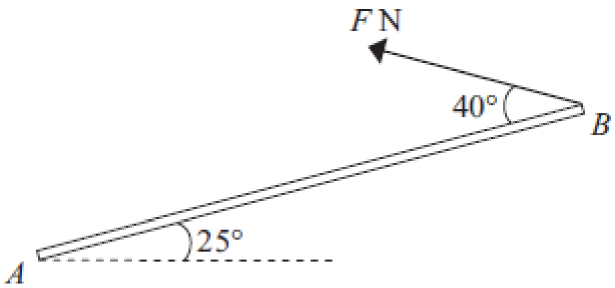

Figure 1

A uniform rod AB, of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

(*a*) Find the value of F.

(4)

(*b*) Find the magnitude and direction of the vertical component of the force acting on the rod at A.

(4)

4.

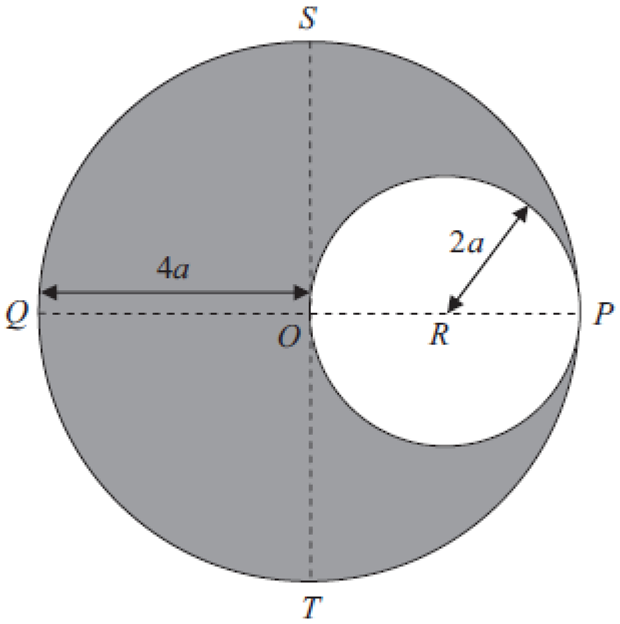

Figure 2

A uniform circular disc has centre O and radius $4a$. The lines PQ and ST are perpendicular diameters of the disc. A circular hole of radius $2a$ is made in the disc, with the centre of the hole at the point R on OP where $OR = 2a$, to form the lamina L, shown shaded in Figure 2.

(*a*) Show that the distance of the centre of mass of L from P is $\dfrac{14a}{3}$.

(4)

The mass of L is m and a particle of mass km is now fixed to L at the point P. The system is now suspended from the point S and hangs freely in equilibrium. The diameter ST makes an angle α with the downward vertical through S, where $\tan \alpha = \frac{5}{6}$.

(*b*) Find the value of k.

(5)

5.

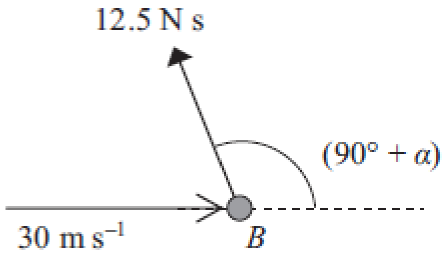

Figure 3

A small ball B of mass 0.25 kg is moving in a straight line with speed 30 m s^{-1} on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 N s and is applied in a horizontal direction making an angle of $(90° + \alpha)$, where $\tan \alpha = \frac{3}{4}$, with the initial direction of motion of the ball, as shown in Figure 3.

(i) Find the speed of B immediately after the impulse is applied.

(ii) Find the direction of motion of B immediately after the impulse is applied.

(6)

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6. A car of mass 1200 kg pulls a trailer of mass 400 kg up a straight road which is inclined to the horizontal at an angle α, where $\sin \alpha = \frac{1}{14}$. The trailer is attached to the car by a light inextensible towbar which is parallel to the road. The car's engine works at a constant rate of 60 kW. The non-gravitational resistances to motion are constant and of magnitude 1000 N on the car and 200 N on the trailer.

At a given instant, the car is moving at 10 m s^{-1}. Find

(*a*) the acceleration of the car at this instant,

(5)

(*b*) the tension in the towbar at this instant.

(4)

The towbar breaks when the car is moving at 12 m s^{-1}.

(*c*) Find, using the work-energy principle, the further distance that the trailer travels before coming instantaneously to rest.

(5)

7.

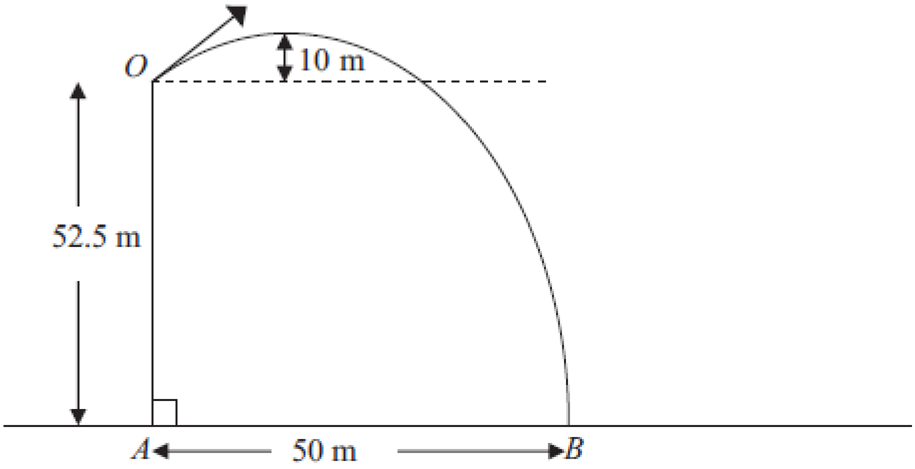

Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where AB = 50 m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(*a*) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.

(3)

(*b*) Find the speed of projection.

(9)

(*c*) Find the time after projection when the stone is moving parallel to OB.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 6t\mathbf{i} + (4 - 2t)\mathbf{j}$	M1 A1
	When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$	DM1
	$\lvert\mathbf{a}\rvert = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32$ (m s^{-2})	DM1 A1
		(5)
(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}\, dt$	M1
	$= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$	A1
	$t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$	DM1
	When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j}$ (m)	DM1 A1
		(5) **10**

Question Number	Scheme	Marks
2. (a)	$3m.2u - 4mu = 3mv_1 + 4mv_2$	M1 A1
	$e(2u + u) = -v_1 + v_2$	M1 A1
	$\frac{u(2+9e)}{7} = v_2$	DM1 A1 (6)
(b)	$v_1 = \frac{2u(1-6e)}{7}$	M1 A1
	$v_1 < 0 \Rightarrow e > \frac{1}{6}$	DM1 A1
	$1 \geq e > \frac{1}{6}$	B1 (5) **11**

Question Number	Scheme	Marks
4. (a)	$\pi(4a)^2 \quad \pi(2a)^2 \quad (\pi(4a)^2 - \pi(2a)^2)$	B1
	$4 \quad 1 \quad 3$	
	$4a \quad 2a \quad \bar{x}$	B1
	$(4 \text{ x } 4a) - (1 \text{ x } 2a) = 3\bar{x}$	M1
	$\frac{14a}{3} = \bar{x}$ *	A1
		(4)
(b)	$OG = 4a\tan\alpha = \frac{10a}{3}\left(\Rightarrow PG = \frac{2a}{3}\right)$	M1
		A1
	$M(P), (m+km)g.\frac{2a}{3}\cos\alpha = mg.\frac{14a}{3}\cos\alpha$	M1
	$M(G): km \times \frac{2}{3}a = m \times \left(\frac{10}{3}a + \frac{2}{3}a\right) = 4ma$	
	$M(O): m(1+k) \times \frac{10}{3}a + m \times \frac{2}{3}a = km \times 4a$	A1
	$M(C): \frac{12}{3}a \times (1+k)m = \frac{14}{3}a \times km$	
	$M(Q): \frac{22}{3}a \times m(1+k) = \frac{10}{3}a \times m + 8a \times km$	
	$k = 6$	A1
		(5)
		(9 marks)

Question Number	Scheme	Marks
5		
	$12.5\sin\alpha = \frac{1}{4}(v_1 - -30)$	M1
	or $-12.5\sin\alpha = \frac{1}{4}\ v_1 - 30$ $\quad v_1 = 0$	A1
	$12.5\cos\alpha = \frac{1}{4}(v_2 - 0)$ $\quad v_2 = 40$	M1 A1
	speed is 40 m s^{-1};	A1
	perpendicular to original direction	A1
		6
OR	Using a vector triangle: $(\frac{1}{4}v)^2 = 7.5^2 + 12.5^2 - 2x7.5x12.5\cos(90^o - \alpha)$	M1 A1
	$v = 40\text{m s}^{-1}$	A1
	$\frac{12.5}{\sin\theta} = \frac{7.5}{\sin\alpha}$	M1 A1
	$\theta = 90^o$	A1
		6

Question Number	Scheme	Marks
6. (a)	$F = \frac{60000}{10} = 6000$	B1
	$F - 1200g\sin\alpha - 400g\sin\alpha - 1000 - 200 = 1600a$	M1 A1 A1
	$a = 2.3$ (m s^{-2})	A1 **(5)**
(b)	$T - 400g\sin\alpha - 200 = 400 \text{ x } 2.3$	M1 A1 ft A1 ft
	$T = 1400$	A1 **(4)**
(c)	$200d = \frac{1}{2}400.12^2 - 400gd\sin\alpha$	M1 A1 A1
	$d = 60$ (m)	DM1 A1 **(5)**
		14 marks

Question Number	Scheme	Marks
7 (a)	$0^2 = u_V{}^2 - 2 \text{ x } 9.8 \text{ x } 10$	M1 A1
	$u_V = 14$ *	A1
		(3)
OR	conservation of energy:	M1
	$\frac{1}{2}m\left(u_h{}^2 + u_v{}^2\right) = mg \times 10 + \frac{1}{2}mu_h{}^2, \frac{1}{2}u_v{}^2 = 98$	A1
	$u_V = 14$ *	A1
		(3)
(b)	$(\uparrow), \; -52.5 = 14t - \frac{1}{2}gt^2$	M1 A1 A1
	$49t^2 - 140t - 525 = 0$	DM1
	$(t-5)(49t+105) = 0 \quad t = 5$	A1
	$(\rightarrow), \quad 50 = 5u_H$	M1
	$u_H = 10$	A1
	$u = \sqrt{10^2 + 14^2}$	M1
	$= \sqrt{296} \approx 17.2 \text{ m s}^{-1}$	A1
		(9)